Final Exam Review: Study Guide

Name:_____

Math 3

Day 1 – Functions, Graphing, Regression

Relation: _____

Function: _____

Domain: _____

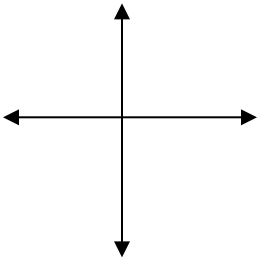
Range: _____

Asymptote: _____

Hole: _____

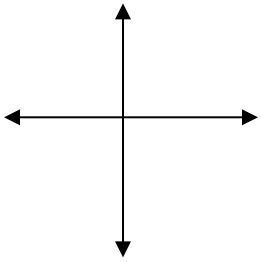
Graphs of Functions

f(x) = $|x|$



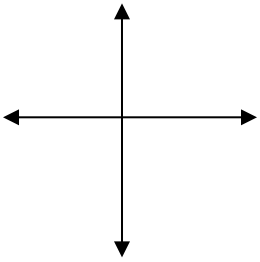
Key Ideas

f(x) = $\sqrt{x}$



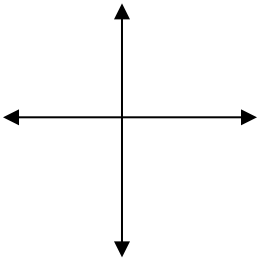
Key Ideas

f(x) = x^2



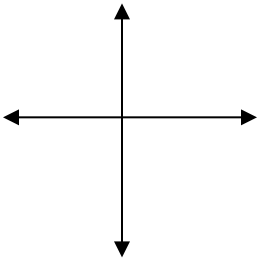
Key Ideas

f(x) = x^3



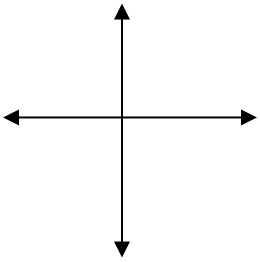
Key Ideas

f(x) = a^x (a is a number)



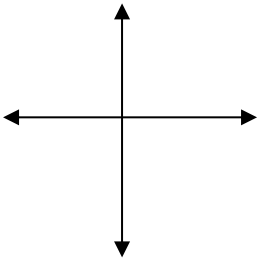
Key Ideas

f(x) = sin x



Key Ideas

f(x) = cos x



Key Ideas

Properties of Graphs

x-intercepts:

y-intercepts:

Translations:

Reflections (Flips):_____

Stretches/Shrinks: If a > 1, _____. If a < 1, _____.

	Inside	**Outside**
Positive (+)		
Negative (–)		

Writing Equations for Functions in the Calculator

Quadratic - _____ Cubic - _____ Exponential - _____ Linear - _____

When a ball is thrown off a 40 foot high roof, it is 53 feet high after 1 second, 28 feet high after 2 seconds, and 5 feet high after 3 seconds. What quadratic equation describes this situation?

Points: _____ Equation: _____

Day 2 – Factoring, Imaginary Numbers, Trigonometry, Exponential Equations

Factoring

GCF $x^2 + bx + c$ $ax^2 + bx + c$ Difference of Perfect Squares

$10x^2 - 5x$ $x^2 - 9x - 22$ $3x^2 - 13x - 10$ $x^2 - 49$ $5x^3 - 500x$

Operations With Imaginary Numbers – Treat *i* like a variable, except i^2 = _____.

Simplify:

1) $(5 + 2i) - (4 + 5i)$ 2) $(3 + 2i)(2 - 7i)$ 3) $\frac{4 + 3i}{5i}$ 4) $\frac{2 - i}{1 - 4i}$

Trigonometry and the Unit Circle

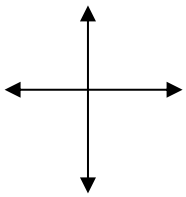
sin = _____ cos = _____ tan = _____

sin 30 = cos 30 = tan 30 =

sin 45 = cos 45 = tan 45 =

sin 60 = cos 60 = tan 60 =

From those, use your sign rules for each quadrant:



The graphs of f(x) = sin x, f(x) = cos x, and f(x) = tan x are _____, as they repeat themselves.

Exponential Equations: **$y = ab^x$**

y = _____ a = _____ b = _____ x = _____

If percent increase, b = _____ (r as a decimal). If percent decrease, b = _____ (r as decimal).

If $0 < b < 1$, the value is _____. If $b > 1$, the value is _____.

If the value compounds CONTINUOUSLY, use $A = Pe^{rt}$.

A = _____ P = _____ e = ____ r = _____ t = _____

For half-life, use: $y = a(.5)^{t/h}$.

y = _____ t = _____ h = _____

Day 3 – Solving Equations

Type	Steps/Things To Remember	Example(s)	
Quadratics	**1. Solve by factoring:** **2. Solve by completing the square:** **3. Solve by Quadratic Formula:**	**1.** $3x^2 - 6x + 8 = 0$	**2.** $x^2 = -10x + 24$
Radicals		**3.** $5\sqrt{3x-9} + 10 = 55$	**4.** $\sqrt[4]{2x+3} = 3$
Rationals		**5.** $\frac{3x}{x+2} - \frac{2x}{x-2} = \frac{10x+5}{x^2-4}$	**6.** $\frac{3}{x+3} + 9 = \frac{10x}{x+3}$

Variables as Exponents (Using Logs)		**7.** $4^{3x} + 8 = 20$	**8.** $6e^{x} = 39$
Variables as Bases (Using Radicals)		**9.** $5x^2 + 10 = 255$	**10.** $x^3 - 10 = -135$
Using *i*		**11.** $x^2 + 50 = 25$	
Absolute Value Equations		**12.** $2\left\|x - 6\right\| - 3 = 15$	

Day 4 – Solving Equations to Solve Word Problems

Quadratics: A rocket was launched to follow the equation: $f(x) = -16x^2 + 250x + 75$.

a) **When** does the rocket reach its maximum height?

b) What is the **maximum height** the rocket reaches?

c) When does the rocket **hit the ground**?

Rational Functions: One volunteer can clean Mr. McDowell's room in 5 hours, and another can clean it in 3.5 hours. How long would it take them to clean it if they worked together?

Exponential Functions: A certain stock's price increases following the function $f(x) = 30(1.035)^x$ after x years.

a) What is the stock's initial price? _____

b) What is the stock's percent increase? _____

c) What would the stock's price be after 9 years?

d) When would the stock's price reach $75?

e) How long would it take the stock's price to double?

f) How long would the stock's price take to triple if it **compounded continuously**?

Higher-Order Equations: Solve: $f(x) = x^4 + 3x^3 + x^2 - 12x - 20$

Day 5 – Graphing Polynomials and Rationals, Statistics, Solving Systems

Polynomial Functions:

Degree: _____

Leading Coefficient: _____

x-intercepts: _____ y-intercept: _____

Example: $f(x) = x^3 - 3x^2 - 4x + 12$

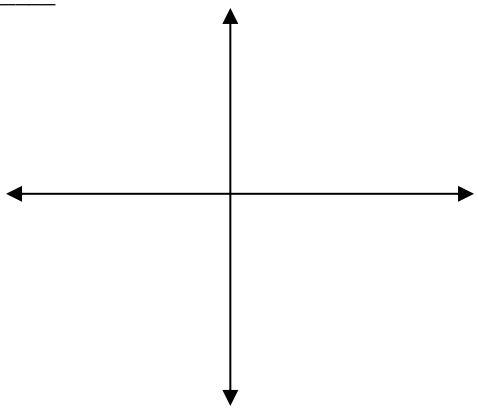
Degree:

LC:

End Behavior:

y-intercept:

x-intercepts:



Rational Functions (Factor First!):

Vertical Asymptotes/Holes: _____

Horizontal Asymptotes: Degree of Numerator Higher: _____ Degree of Denominator Higher: _____

Degree of Numerator and Denominator Equal: _____

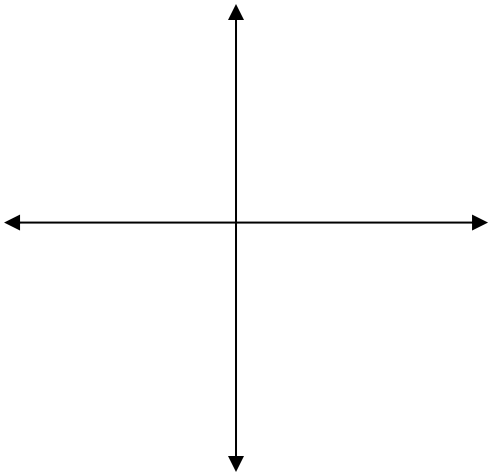
Example: $f(x) = \frac{2x^2 + 7x - 15}{2x^2 + x - 6}$

VA –

Holes –

HA –

x-int: y-int:



Statistics and the Normal Curve

Mean: _____

Standard Deviation: _____

_____ of the data is within 1 SD of the mean, _____ is within 2 SD, _____ is within 3 SD.

Example: If data has a mean of 50 with a standard deviation of 3:

- 68% of the data is between _____, 95% of the data is between _____, and 99.7% of the data is between _____.

Solving Systems Using the Calculator

-Make sure both equations are solved for y and put them in Y1= and Y2=.

-2nd-Trace-5→enter, enter, enter for each intersection

-The **ordered pair** (x and y) is the answer.

Day 6 - Operations with Rational Expressions:

Domain Restrictions for Rational Expressions: _____

Multiplying Rationals – Factor first, then _____

1. $\frac{x^2+3x+2}{x-1} \cdot \frac{1-x}{x+2}$

2. $\frac{2x^2+5x-3}{x^2-4x} \cdot \frac{2x^3-8x^2}{x^2+6x+9}$

Dividing Rationals – Factor first, then _____ or _____

1. $\frac{x^2-2x-8}{x+3} \div \frac{x-4}{x+3}$

Adding or Subtracting Rationals – Factor first, then _____

1. $\frac{8}{x^2-25} + \frac{9}{x-5}$

2. $\frac{x-3}{x^2+3x} + \frac{7}{x+3}$

Practice Problems

1. Solve the equation. Check for extraneous solutions. $4|5 - 5x| = 7x + 6$

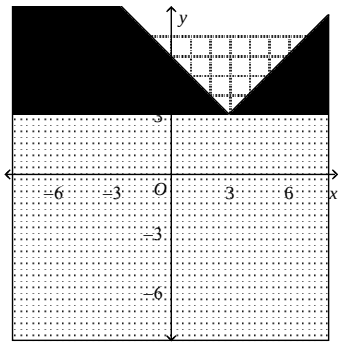
   a. $x = 2$ or $x = \frac{14}{13}$

   b. $x = \frac{14}{27}$

   c. $x = 2$

   d. $x = 2$ or $x = \frac{14}{27}$

2. Write an inequality for the graph.



   a. $y \geq |x - 3| + 3$

   b. $y \leq |x - 3| + 3$

   c. $y \leq |x + 3| + 3$

   d. $y \leq |x - 3| - 3$

3. Solve the system $\begin{cases} -0.5x - y = 3.5 \\ 3.25x - y = 7.25 \end{cases}$

   a. (–4, 1)

   b. (–1, 4)

   c. (4, –1)

   d. (1, –4)

4. Solve the system $\begin{cases} -2x + 3y + 3z = -6 \\ -x + 2y + 2z = -1 \\ x + 5z = -6 \end{cases}$

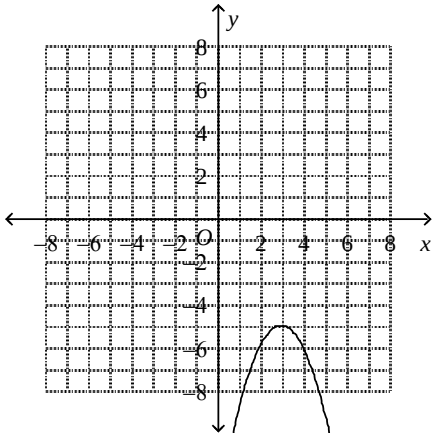
   a. (–9, 7, –3)

   b. (9, 7, –3)

   c. (9, 7, 3)

   d. (9, –7, –3)

5. Use the vertex form to write the equation of the parabola.



   a. $y = -(x + 3)^2 + 5$

   b. $y = -(x + 3)^2 - 5$

   c. $y = (x - 3)^2 - 5$

   d. $y = -(x - 3)^2 - 5$

6. What are the solutions of the quadratic equation? $2x^2 + 25x + 72 = 0$

   a. –8, 2

   b. $-8, -\frac{9}{2}$

   c. $-\frac{9}{2}, -\frac{1}{4}$

   d. $8, -\frac{1}{4}$

7. Solve. $5x^2 - 5x = 9$

   a. $-\frac{1}{2} \pm \frac{\sqrt{40}}{10}$

   b. $1 \pm \frac{\sqrt{175}}{5}$

   c. $-1 \pm \frac{\sqrt{205}}{5}$

   d. $\frac{1}{2} \pm \frac{\sqrt{205}}{10}$

8. Use the Quadratic Formula to solve the equation. $5x^2 - 9x - 8 = 0$

a. $\frac{9}{10} \pm \frac{\sqrt{241}}{10}$
c. $\frac{10}{9} \pm \frac{\sqrt{482}}{10}$
b. $\frac{9}{5} \pm \frac{\sqrt{241}}{10}$
d. $\frac{9}{5} \pm \frac{\sqrt{241}}{5}$

9. Simplify the number using the imaginary unit *i*. $\sqrt{-75}$

a. $i\sqrt{75}$
c. $-5\sqrt{3}$
b. $5\sqrt{-3}$
d. $5i\sqrt{3}$

10. Simplify the expression. $\frac{-5-4i}{6+2i}$

a. $\frac{-38-14i}{40}$
c. $\frac{-38-14i}{32}$
b. $\frac{-22-14i}{40}$
d. $\frac{-38+34i}{40}$

$\frac{1}{2}x^2 + x + 4 = 0$

11. What are the solutions?

a. $-1 \pm \sqrt{7}i$
c. $1 \pm \sqrt{7}i$
b. $1 \pm \sqrt{7}i$
d. $-1 \pm \sqrt{7}i$ |

12. What is a quartic polynomial function in standard form with zeros 1, –1, –3, and 2?

a. $g(x) = x^4 + x^3 - 4x^2 - 7x + 6$
c. $g(x) = x^4 + x^3 - 7x^2 - x + 6$
b. $g(x) = x^4 - x^3 + 7x^2 + x + 6$
d. $g(x) = x^4 - x^3 - 4x^2 + x + 6$

13. Determine which binomial is *not* a factor of $4x^4 - 21x^3 - 46x^2 + 219x + 180$.

a. x – 5
c. x – 6
b. x + 3
d. 4x + 3

14. Find a third-degree polynomial equation with rational coefficients that has roots –3 and 4 + *i*.

a. $x^3 - 5x^2 - 7x = 0$
c. $x^3 - 5x^2 - 7x + 51 = 0$
b. $x^3 - 8x^2 + 17x = 0$
d. $x^3 - 5x^2 - 8x + 17 = 0$

15. Find a quadratic equation with roots –1 + 4*i* and –1 – 4*i*.

a. $x^2 - 2x + 17 = 0$
c. $x^2 + 2x + 17 = 0$
b. $x^2 + 2x - 17 = 0$
d. $x^2 - 2x - 17 = 0$

16. What does Descartes' Rule of Signs tell you about the real roots of the polynomial? $x^3 + 3x^2 - 4x - 6 = 0$

a. There are either 2 or 0 positive roots and one negative root.
b. There are either 2 or 0 positive roots and there are either 2 or 0 negative roots.
c. There is one positive root and either 2 or 0 negative roots.
d. There is one positive root and one negative root.

17. Find all the zeros of the equation. $3x^5 - 3x^4 - 72x^3 + 72x^2 - 75x + 75 = 0$

a. 1, –5, $-i$
c. 1, 5, *i*
b. 5, –5, *i*, $-i$
d. 1, 5, –5, *i*, $-i$

18. Use Pascal's Triangle to expand the binomial. $(2v + s)^5$

a. $s^5 + 20s^4v + 80s^3v^2 + 160s^2v^3 + 160sv^4 + 64v^5$
b. $s^5 - 5s^4v + 10s^3v^2 - 10s^2v^3 + 5sv^4 - v^5$
c. $s^5 + 10s^4v + 40s^3v^2 + 80s^2v^3 + 80sv^4 + 32v^5$
d. $s^5 + 10s^4 + 40s^3 + 80s^2 + 80s + 32$

19. What is the equation of $y = x^3$ with the given transformations? vertical compression by a factor of $\frac{1}{5}$, horizontal shift 5 units to the left, reflection across the x-axis

a. $y = -\frac{1}{5}(x)^3 + 5$

c. $y = \frac{1}{5}(x+5)^3$

b. $y = -\frac{1}{5}(x+5)^3$

d. $y = -\frac{1}{5}(x-5)^3$

20. What is the solution of the equation? $3 + \sqrt{x-4} = 10$

a. 11 b. 49 c. 45 d. 53

21. What is the solution of the equation? $4(3-x)^{\frac{4}{3}} - 5 = 59$

a. –5, 11 b. 5 c. 11 d. –11

22. What is the solution of the equation? $\sqrt{2x+20} - 6 = x$

a. –2 b. –8 and –2 c. 8 and –2 d. –8

23. Suppose you invest $1500 at an annual interest rate of 3.4% compounded continuously. How much will you have in the account after 30 years?

a. $46,556.31 b. $4,159.79 c. $4,158.97 d. $63,222.64

24. Write the equation in logarithmic form. $7^2 = 49$

a. $\log_2 49 = 7$

c. $\log_7 49 = 2$

b. $\log 49 = 2 \cdot 7$

d. $\log 49 = 2$

25. Expand the logarithmic expression. $\log_b \sqrt{\frac{57}{74}}$

a. $\frac{1}{2}\log_b 57 + \frac{1}{2}\log_b 74$

c. $\sqrt{\log_b 57 - \log_b 74}$

b. $\frac{1}{2}\log_b 57 - \frac{1}{2}\log_b 74$

d. $\log_b \frac{1}{2}(57 - 74)$

26 Solve the logarithmic equation. $\log(x+9) - \log x = 3$

a. 0.0090 b. 0.3103 c. 3.2222 d. 111

27. Use natural logarithms to solve the equation. $8e^{2x} + 4 = 18$

a. 0.165 b. 0.506 c. 0.280 d. –1.595

28. Simplify the rational expression. State any restrictions on the variable. $\frac{z^4 - 12z^2 + 35}{z^4 - 9z^2 + 14}$

a. $\frac{z^2 - 5}{z^2 - 2}; z \neq \pm\sqrt{7}, z \neq \pm\sqrt{2}$

c. $\frac{z^2 - 5}{z^2 - 2}; z \neq 7, z \neq 2$

b. $\frac{-(z^2 - 5)}{z^2 - 2}; z \neq \pm\sqrt{7}, z \neq \pm\sqrt{2}$

d. $\frac{z^2 - 5}{z^2 - 2}; z \neq 7, z \neq -2$

29. Simplify the sum. $\frac{m^2 - 6m + 5}{m^2 - 10m + 25} + \frac{4}{m-5}$

a. $\frac{m^2 - 6m + 9}{m^2 - 10m + 25}$ b. $m + 3$ c. $\frac{m+3}{m-5}$ d. $\frac{m-1}{m-5}$

30. Simplify the complex fraction. $\dfrac{\frac{4}{2p} - \frac{1}{3p}}{\frac{1}{2p} + \frac{4}{5p}}$

a. $\frac{21}{25}$ b. $\frac{39}{50}$ c. $\frac{50}{39}$ d. $\frac{25}{21}$

31. Solve the equation. Check the solution. $\frac{7}{3z} + \frac{6}{z} = -4$

a. $-\frac{9}{4}$ b. $\frac{25}{3}$ c. $-\frac{13}{16}$ d. $-\frac{25}{12}$

32. A group of college students are volunteering for Help the Homeless during their spring break. They are putting the finishing touches on a house they built. Working alone, Jorge can paint a certain room in 10 hours. Maria can paint the same room in 8 hours. Write an equation that can be used to find how long it will take them working together to paint the room. How many hours will it take them to paint the room? If necessary, round your answer to the nearest hundredth.

a. $\frac{x}{8} + \frac{x}{10} = 1$; 9 hours
b. $\frac{x}{10} + \frac{x}{8} = 1$; 4.44 hours
c. $\frac{10}{x} + \frac{8}{x} = 1$; 9 hours
d. $\frac{10}{x} + \frac{8}{x} = 1$; 18 hours

33. Which of the following angles is not coterminal with the other three?

a. 502° b. 142° c. 38° d. –218°

34. Find the exact values of cos 330° and sin 330°.

a. $\cos = -\frac{\sqrt{3}}{2}, \sin = \frac{1}{2}$
b. $\cos = \frac{\sqrt{3}}{2}, \sin = -\frac{1}{2}$
c. $\cos = -\frac{1}{2}, \sin = \frac{\sqrt{3}}{2}$
d. $\cos = \frac{1}{2}, \sin = -\frac{\sqrt{3}}{2}$

35. Find the radian measure of an angle of –310°.

a. $\frac{-31}{18\pi}$ b. $\frac{-31\pi}{18}$ c. $\frac{18\pi}{-31}$ d. $\frac{18}{-31\pi}$

36. Find the degree measure of an angle of $\frac{3\pi}{2}$ radians.

a. 4.71° b. $270\pi°$ c. $\frac{\pi}{120}°$ d. 270°

37. Find the exact values of $\cos\left(\frac{3\pi}{4} \text{ radians}\right)$ and $\sin\left(\frac{3\pi}{4} \text{ radians}\right)$.

a. $\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}$ b. $-\frac{1}{2}, \frac{\sqrt{3}}{2}$ c. $-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$ d. $-\frac{\sqrt{3}}{2}, \frac{1}{2}$

38. What is the value of the expression? $\tan\frac{4\pi}{3}$

a. 1 b. $-\sqrt{3}$ c. $\frac{1}{\sqrt{3}}$ d. $\sqrt{3}$

39. Find the exact value. csc 45°

a. 0 b. undefined c. $\frac{1}{2}$ d. $\sqrt{2}$

Free Response

40. Suppose a parabola has vertex (–4, 1) and also passes through the point (–3, 2). Write the equation of the parabola in vertex form.

41. A historian took a count of the number of people in a Gold Rush town for six years in the 1870's. Find a quadratic function that models the data as a function of x, the number of years since 1870. Use the model to estimate the number of people in the town in 1877.

Year	1870	1871	1872	1873	1874	1875	1876
Population	460	480	488	484	468	440	400

42. What is the expression in factored form? $-2x^2 - 2x + 24$

43. What is the expression in factored form? $2x^2 + 29x + 90$

44. What is the end behavior of the graph? $-4x^5 + 4x^4 - 5x^3 - 8$

45. Find the roots of the polynomial equation. $x^3 - 13x^2 + 57x - 105 = 0$

46. What polynomial has a graph that passes through these points? (–3, 133), (–1, 1), (0, 1), (3, 49), (4, 161)

47. What is the inverse of the given relation? $y = 3x^2 - 2$.

48. Evaluate the logarithm. $\log_3 \frac{1}{9}$

49. Write the expression as a single logarithm. $4 \log_b q + 8 \log_b t$

50. What is the value of $\log_8 2$?

51. Solve ln x – ln 12 = 1.

52. What is the quotient in simplified form? State any restrictions on the variable. $\frac{z+5}{z-4} \div \frac{z+6}{z^2 - 7z + 12}$

53. Solve the exponential equation. $50^{3x+1} = 75$